

6-1 Study Guide and Intervention *(continued)*

Solving Inequalities by Addition and Subtraction

Solve Inequalities by Subtraction Subtraction can be used to solve inequalities. If any number is subtracted from each side of a true inequality, the resulting inequality is also true.

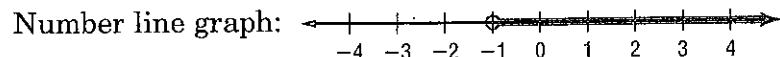
Subtraction Property of Inequalities	For all numbers a , b , and c , if $a > b$, then $a - c > b - c$, and if $a < b$, then $a - c < b - c$.
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The property is also true when $>$ and $<$ are replaced with \geq and \leq .

Example Solve $3a + 5 > 4 + 2a$. Then graph it on a number line.

$3a + 5 > 4 + 2a$	Original inequality
$3a + 5 - 2a > 4 + 2a - 2a$	Subtract $2a$ from each side.
$a + 5 > 4$	Simplify.
$a + 5 - 5 > 4 - 5$	Subtract 5 from each side.
$a > -1$	Simplify.

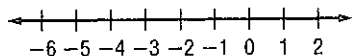
The solution is $\{a \mid a > -1\}$.



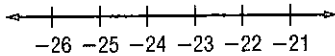
Exercises

Solve each inequality. Then check your solution, and graph it on a number line.

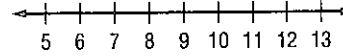
1. $t + 12 \geq 8$



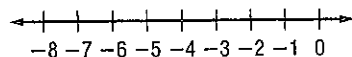
2. $n + 12 > -12$



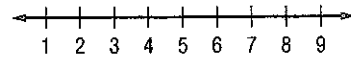
3. $16 \leq h + 9$



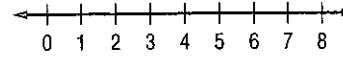
4. $y + 4 > -2$



5. $3r + 6 > 4r$



6. $\frac{3}{2}q - 5 \geq \frac{1}{2}q$



Solve each inequality. Then check your solution.

7. $4p \geq 3p + 0.7$

8. $r + \frac{1}{4} > \frac{3}{8}$

9. $9k + 12 > 8k$

10. $-1.2 > 2.4 + y$

11. $4y < 5y + 14$

12. $3n + 17 < 4n$

Define a variable, write an inequality, and solve each problem. Then check your solution.

13. The sum of a number and 8 is less than 12.

14. The sum of two numbers is at most 6, and one of the number is -2 .

15. The sum of a number and 6 is greater than or equal to -4 .

6-1

Practice

Solving Inequalities by Addition and Subtraction

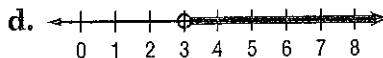
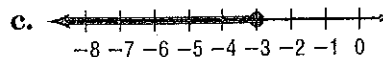
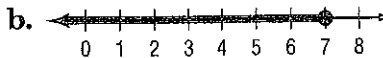
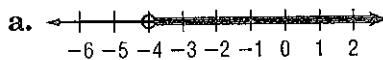
Match each inequality with its corresponding graph.

1. $-8 \geq x - 15$

2. $4x + 3 < 5x$

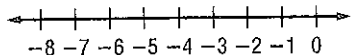
3. $8x > 7x - 4$

4. $12 + x \leq 9$

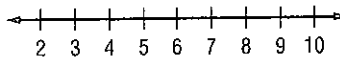


Solve each inequality. Then check your solution, and graph it on a number line.

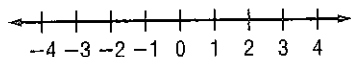
5. $r - (-5) > -2$



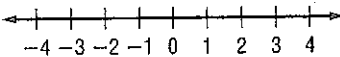
6. $3x + 8 \geq 4x$



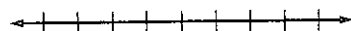
7. $n - 2.5 \geq -5$



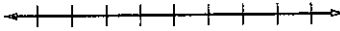
8. $1.5 < y + 1$



9. $z + 3 > \frac{2}{3}$



10. $\frac{1}{2} \leq c - \frac{3}{4}$



Define a variable, write an inequality, and solve each problem. Then check your solution.

11. The sum of a number and 17 is no less than 26.

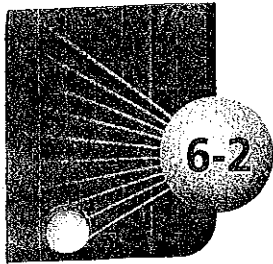
12. Twice a number minus 4 is less than three times the number.

13. Twelve is at most a number decreased by 7.

14. Eight plus four times a number is greater than five times the number.

15. **ATMOSPHERIC SCIENCE** The troposphere extends from the earth's surface to a height of 6–12 miles, depending on the location and the season. If a plane is flying at an altitude of 5.8 miles, and the troposphere is 8.6 miles deep in that area, how much higher can the plane go without leaving the troposphere?

16. **EARTH SCIENCE** Mature soil is composed of three layers, the uppermost being topsoil. Jamal is planting a bush that needs a hole 18 centimeters deep for the roots. The instructions suggest an additional 8 centimeters depth for a cushion. If Jamal wants to add even more cushion, and the topsoil in his yard is 30 centimeters deep, how much more cushion can he add and still remain in the topsoil layer?



6-2 Study Guide and Intervention

Solving Inequalities by Multiplication and Division

Solve Inequalities by Multiplication If each side of an inequality is multiplied by the same positive number, the resulting inequality is also true. However, if each side of an inequality is multiplied by the same negative number, the direction of the inequality must be reversed for the resulting inequality to be true.

Multiplication Property of Inequalities	For all numbers a , b , and c , with $c \neq 0$, 1. if c is positive and $a > b$, then $ac > bc$; if c is positive and $a < b$, then $ac < bc$; 2. if c is negative and $a > b$, then $ac < bc$; if c is negative and $a < b$, then $ac > bc$.
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The property is also true when $>$ and $<$ are replaced with \geq and \leq .

Example 1 Solve $-\frac{y}{8} \geq 12$.

$$-\frac{y}{8} \geq 12 \quad \text{Original equation}$$

$$(-8)\left(-\frac{y}{8}\right) \leq (-8)12 \quad \text{Multiply each side by } -8; \text{ change } \geq \text{ to } \leq.$$

$$y \leq -96 \quad \text{Simplify.}$$

The solution is $\{y \mid y \leq -96\}$.

Example 2 Solve $\frac{3}{4}k < 15$.

$$\frac{3}{4}k < 15 \quad \text{Original equation}$$

$$\left(\frac{4}{3}\right)\frac{3}{4}k < \left(\frac{4}{3}\right)15 \quad \text{Multiply each side by } \frac{4}{3}.$$

$$k < 20 \quad \text{Simplify.}$$

The solution is $\{k \mid k < 20\}$.

Exercises

Solve each inequality. Then check your solution.

1. $\frac{y}{6} \leq 2$

2. $-\frac{n}{50} > 22$

3. $\frac{3}{5}h \geq -3$

4. $-\frac{p}{6} < -6$

5. $\frac{1}{4}n \geq 10$

6. $-\frac{2}{3}b < \frac{1}{3}$

7. $\frac{3m}{5} < -\frac{3}{20}$

8. $-2.51 \leq -\frac{2h}{4}$

9. $\frac{g}{5} \geq -2$

10. $-\frac{3}{4} > -\frac{9p}{5}$

11. $\frac{n}{10} \geq 5.4$

12. $\frac{2a}{7} \geq -6$

Define a variable, write an inequality, and solve each problem. Then check your solution.

13. Half of a number is at least 14.

14. The opposite of one-third a number is greater than 9.

15. One fifth of a number is at most 30.

Lesson 6-2

6-2

Practice

Solving Inequalities by Multiplication and Division

Match each inequality with its corresponding statement.

- | | |
|--------------------------|---|
| 1. $-4n \geq 5$ | a. Negative four times a number is less than five. |
| 2. $\frac{4}{5}n > 5$ | b. Four fifths of a number is no more than five. |
| 3. $4n \leq 5$ | c. Four times a number is fewer than five. |
| 4. $\frac{4}{5}n \leq 5$ | d. Negative four times a number is no less than five. |
| 5. $4n < 5$ | e. Four times a number is at most five. |
| 6. $-4n < 5$ | f. Four fifths of a number is more than five. |

Solve each inequality. Then check your solution.

- | | | | |
|--------------------------|--------------------------|-----------------------------|------------------------------|
| 7. $-\frac{a}{5} < -14$ | 8. $-13h \leq 52$ | 9. $\frac{s}{16} \geq -6$ | 10. $39 > 13p$ |
| 11. $\frac{2}{3}n > -12$ | 12. $-\frac{5}{9}t < 25$ | 13. $-\frac{3}{5}m \leq -6$ | 14. $\frac{10}{3}k \geq -10$ |
| 15. $-3b \leq 0.75$ | 16. $-0.9c > -9$ | 17. $0.1x \geq -4$ | 18. $-2.3 < \frac{j}{4}$ |
| 19. $-15y < 3$ | 20. $2.6v \geq -20.8$ | 21. $0 > -0.5u$ | 22. $\frac{7}{8}f \leq -1$ |

Define a variable, write an inequality, and solve each problem. Then check your solution.

23. Negative three times a number is at least 57.
24. Two thirds of a number is no more than -10 .
25. Negative three fifths of a number is less than -6 .
26. **FLOODING** A river is rising at a rate of 3 inches per hour. If the river rises more than 2 feet, it will exceed flood stage. How long can the river rise at this rate without exceeding flood stage?
27. **SALES** Pet Supplies makes a profit of \$5.50 per bag on its line of natural dog food. If the store wants to make a profit of no less than \$5225, how many bags of dog food does it need to sell?

6-3 Study Guide and Intervention

Solving Multi-Step Inequalities

Solve Multi-Step Inequalities To solve linear inequalities involving more than one operation, undo the operations in reverse of the order of operations, just as you would solve an equation with more than one operation.

Example 1 Solve $6x - 4 \leq 2x + 12$.

$$\begin{array}{ll}
 6x - 4 \leq 2x + 12 & \text{Original inequality} \\
 6x - 4 - 2x \leq 2x + 12 - 2x & \text{Subtract } 2x \text{ from} \\
 & \text{each side.} \\
 4x - 4 \leq 12 & \text{Simplify.} \\
 4x - 4 + 4 \leq 12 + 4 & \text{Add 4 to each side.} \\
 4x \leq 16 & \text{Simplify.} \\
 \frac{4x}{4} \leq \frac{16}{4} & \text{Divide each side by 4.} \\
 x \leq 4 & \text{Simplify.}
 \end{array}$$

The solution is $\{x \mid x \leq 4\}$.

Example 2 Solve $3a - 15 > 4 + 5a$.

$$\begin{array}{ll}
 3a - 15 > 4 + 5a & \text{Original inequality} \\
 3a - 15 - 5a > 4 + 5a - 5a & \text{Subtract } 5a \text{ from} \\
 & \text{each side.} \\
 -2a - 15 > 4 & \text{Simplify.} \\
 -2a - 15 + 15 > 4 + 15 & \text{Add 15 to each side.} \\
 -2a > 19 & \text{Simplify.} \\
 \frac{-2a}{-2} < \frac{19}{-2} & \text{Divide each side by } -2 \\
 & \text{and change } > \text{ to } < . \\
 a < -9\frac{1}{2} & \text{Simplify.}
 \end{array}$$

The solution is $\left\{a \mid a < -9\frac{1}{2}\right\}$.

Exercises

Solve each inequality. Then check your solution.

1. $11y + 13 \geq -1$

2. $8n - 10 < 6 - 2n$

3. $\frac{q}{7} + 1 > -5$

4. $6n + 12 < 8 + 8n$

5. $-12 - d > -12 + 4d$

6. $5r - 6 > 8r - 18$

7. $\frac{-3x + 6}{2} \leq 12$

8. $7.3y - 14.4 > 4.9y$

9. $-8m - 3 < 18 - m$

10. $-4y - 10 > 19 - 2y$

11. $9n - 24n + 45 > 0$

12. $\frac{4x - 2}{5} \geq -4$

Define a variable, write an inequality, and solve each problem. Then check your solution.

13. Negative three times a number plus four is no more than the number minus eight.

14. One fourth of a number decreased by three is at least two.

15. The sum of twelve and a number is no greater than the sum of twice the number and -8 .

6-3

Practice

Solving Multi-Step Inequalities

Justify each indicated step.

1. $x > \frac{5x - 12}{8}$

$8x > (8) \frac{5x - 12}{8}$ a. ?

$8x > 5x - 12$

$8x - 5x > 5x - 12 - 5x$ b. ?

$3x > -12$

$\frac{3x}{3} > \frac{-12}{3}$ c. ?

$x > -4$

2. $2(2h + 2) < 2(3h + 5) - 12$

$4h + 4 < 6h + 10 - 12$ a. ?

$4h + 4 < 6h - 2$

$4h + 4 - 6h < 6h - 2 - 6h$ b. ?

$-2h + 4 < -2$

$-2h + 4 - 4 < -2 - 4$ c. ?

$-2h < -6$

$\frac{-2h}{-2} > \frac{-6}{-2}$ d. ?

$h > 3$

Solve each inequality. Then check your solution.

3. $-5 - \frac{t}{6} \geq -9$

4. $4u - 6 \geq 6u - 20$

5. $13 > \frac{2}{3}a - 1$

6. $\frac{w + 3}{2} < -8$

7. $\frac{3f - 10}{5} > 7$

8. $h \leq \frac{6h + 3}{5}$

9. $3(z + 1) + 11 < -2(z + 13)$

10. $3e + 2(4e + 2) \leq 2(6e + 1)$

11. $5n - 3(n - 6) \geq 0$

Define a variable, write an inequality, and solve each problem. Then check your solution.

12. A number is less than one fourth the sum of three times the number and four.

13. Two times the sum of a number and four is no more than three times the sum of the number and seven decreased by four.

14. **GEOMETRY** The area of a triangular garden can be no more than 120 square feet. The base of the triangle is 16 feet. What is the height of the triangle?15. **MUSIC PRACTICE** Nabuko practices the violin at least 12 hours per week. She practices for three fourths of an hour each session. If Nabuko has already practiced 3 hours in one week, how many sessions remain to meet or exceed her weekly practice goal?