

9-5

Study Guide and Intervention

Factoring Differences of Squares

Factor $a^2 - b^2$ The binomial expression $a^2 - b^2$ is called the **difference of two squares**. The following pattern shows how to factor the difference of squares.

Difference of Squares	$a^2 - b^2 = (a - b)(a + b) = (a + b)(a - b)$.
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Example 1

Factor each binomial.

a. $n^2 - 64$

$$\begin{aligned} n^2 - 64 &= n^2 - 8^2 && \text{Write in the form } a^2 - b^2. \\ &= (n + 8)(n - 8) && \text{Factor.} \end{aligned}$$

b. $4m^2 - 81n^2$

$$\begin{aligned} 4m^2 - 81n^2 &= (2m)^2 - (9n)^2 && \text{Write in the form } a^2 - b^2. \\ &= (2m - 9n)(2m + 9n) && \text{Factor.} \end{aligned}$$

Example 2

Factor each polynomial.

a. $50a^2 - 72$

$$\begin{aligned} 50a^2 - 72 &= 2(25a^2 - 36) && \text{Find the GCF.} \\ &= 2[(5a)^2 - 6^2] && 25a^2 = 5a \cdot 5a \text{ and } 36 = 6 \cdot 6 \\ &= 2(5a + 6)(5a - 6) && \text{Factor the difference of squares.} \end{aligned}$$

b. $4x^4 + 8x^3 - 4x^2 - 8x$

$$\begin{aligned} 4x^4 + 8x^3 - 4x^2 - 8x & \text{Original polynomial} \\ &= 4x(x^3 + 2x^2 - x - 2) && \text{Find the GCF.} \\ &= 4x[(x^3 + 2x^2) - (x + 2)] && \text{Group terms.} \\ &= 4x[x^2(x + 2) - 1(x + 2)] && \text{Find the GCF.} \\ &= 4x[(x^2 - 1)(x + 2)] && \text{Factor by grouping.} \\ &= 4x[(x - 1)(x + 1)(x + 2)] && \text{Factor the difference of squares.} \end{aligned}$$

Exercises

Factor each polynomial if possible. If the polynomial cannot be factored, write *prime*.

1. $x^2 - 81$

2. $m^2 - 100$

3. $16n^2 - 25$

4. $36x^2 - 100y^2$

5. $49x^2 - 32$

6. $16a^2 - 9b^2$

7. $225c^2 - a^2$

8. $72p^2 - 50$

9. $-2 + 2x^2$

10. $-81 + a^4$

11. $6 - 54a^2$

12. $8y^2 - 200$

13. $4x^3 - 100x$

14. $2y^4 - 32y^2$

15. $8m^3 - 128m$

16. $6x^2 - 25$

17. $2a^3 - 98ab^2$

18. $18y^2 - 72y^4$

19. $169x^3 - x$

20. $3a^4 - 3a^2$

21. $3x^4 + 6x^3 - 3x^2 - 6x$

9-5 Study Guide and Intervention *(continued)*

Factoring Differences of Squares

Solve Equations by Factoring Factoring and the Zero Product Property can be used to solve equations that can be written as the product of any number of factors set equal to 0.

Example Solve each equation. Check your solutions.

a. $x^2 - \frac{1}{25} = 0$

$$x^2 - \frac{1}{25} = 0 \quad \text{Original equation}$$

$$x^2 - \left(\frac{1}{5}\right)^2 = 0 \quad x^2 = x \cdot x \text{ and } \frac{1}{25} = \left(\frac{1}{5}\right)\left(\frac{1}{5}\right)$$

$$\left(x + \frac{1}{5}\right)\left(x - \frac{1}{5}\right) = 0 \quad \text{Factor the difference of squares.}$$

$$x + \frac{1}{5} = 0 \quad \text{or} \quad x - \frac{1}{5} = 0 \quad \text{Zero Product Property}$$

$$x = -\frac{1}{5} \quad x = \frac{1}{5} \quad \text{Solve each equation.}$$

The solution set is $\left\{-\frac{1}{5}, \frac{1}{5}\right\}$. Since $\left(-\frac{1}{5}\right)^2 - \frac{1}{25} = 0$ and $\left(\frac{1}{5}\right)^2 - \frac{1}{25} = 0$, the solutions check.

b. $4x^3 = 9x$

$$4x^3 = 9x \quad \text{Original equation}$$

$$4x^3 - 9x = 0 \quad \text{Subtract } 9x \text{ from each side.}$$

$$x(4x^2 - 9) = 0 \quad \text{Find the GCF.}$$

$$x[(2x)^2 - 3^2] = 0 \quad 4x^2 = 2x \cdot 2x \text{ and } 9 = 3 \cdot 3$$

$$x[(2x)^2 - 3^2] = x[(2x - 3)(2x + 3)] \quad \text{Factor the difference of squares.}$$

$$x = 0 \quad \text{or} \quad (2x - 3) = 0 \quad \text{or} \quad (2x + 3) = 0 \quad \text{Zero Product Property}$$

$$x = 0 \quad x = \frac{3}{2} \quad x = -\frac{3}{2} \quad \text{Solve each equation.}$$

The solution set is $\left\{0, \frac{3}{2}, -\frac{3}{2}\right\}$.

Since $4(0)^3 = 9(0)$, $4\left(\frac{3}{2}\right)^3 = 9\left(\frac{3}{2}\right)$, and $4\left(-\frac{3}{2}\right)^3 = 9\left(-\frac{3}{2}\right)$, the solutions check.

Exercises

Solve each equation. Check your solutions.

1. $81x^2 = 49$

2. $36n^2 = 1$

3. $25d^2 - 100 = 0$

4. $\frac{1}{4}x^2 = 25$

5. $36 = \frac{1}{25}x^2$

6. $\frac{49}{100} - x^2 = 0$

7. $9x^3 = 25x$

8. $7a^3 = 175a$

9. $2m^3 = 32m$

10. $16y^3 = 25y$

11. $\frac{1}{64}x^2 = 49$

12. $4a^3 - 64a = 0$

13. $3b^3 - 27b = 0$

14. $\frac{9}{25}m^2 = 121$

15. $48n^3 = 147n$